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COLLEGE OF ENGINEERING AND TECHNOLOGY

(An ISO 9001:2015 Certified Institution)
Anguchettypalayam, Panruti – 607106.

QUESTION BANK (R-2017)

MA8353
TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATION

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QUESTION BANK

PERIOD: JULY - NOV 2020 **BATCH**: 2018 – 2022

BRANCH: EEE **YEAR/SEM:** II/03

SUB CODE/NAME: MA8353 - TRANSFORM AND PARTIAL DIFFERENTIAL EQUATIONS

UNIT I - PARTIAL DIFFERENTIAL EQUATIONS

PART-B

FIRST HALF (8-MARKS)

I - Lagrange's method

1. Solve:- $(mz - ny)p + (nx - lz)q = (ly - mx)$
2. Solve:- $x(\boldsymbol{y} - \boldsymbol{z})\boldsymbol{p} + y(\boldsymbol{z} - \boldsymbol{x})\boldsymbol{q} = z(\boldsymbol{x} - \boldsymbol{y})$ **(A/M-18) (N/D-14)**
3. Solve:- $x^2(\boldsymbol{y} - \boldsymbol{z})\boldsymbol{p} + y^2(\boldsymbol{z} - \boldsymbol{x})\boldsymbol{q} = z^2(\boldsymbol{x} - \boldsymbol{y})$ **(A/M-17) (N/D-19)**
4. **Solve:-** $x(y^2 - z^2)\boldsymbol{p} - y(\boldsymbol{z^2} - x^2)\boldsymbol{q} = z(\boldsymbol{x^2} - \boldsymbol{y^2})$
5. **Solve:-** $x(y^2 + z)\boldsymbol{p} - y(x^2 + z)\boldsymbol{q} = z(\boldsymbol{x^2} - \boldsymbol{y^2})$
6. **Solve:-** $x(z^2 - y^2)\boldsymbol{p} - y(\boldsymbol{x^2} - z^2)\boldsymbol{q} = z(\boldsymbol{y^2} - \boldsymbol{x^2})$ **(N/D-16)**
7. Solve:- $(\boldsymbol{x} - \boldsymbol{2z})\boldsymbol{p} + y(\boldsymbol{2z} - \boldsymbol{y})\boldsymbol{q} = (\boldsymbol{y} - \boldsymbol{x})$ **(N/D-17)**
8. **Solve:-** $(x^2 - yz)\boldsymbol{p} + (y^2 - zx)\boldsymbol{q} = (\boldsymbol{z^2} - \boldsymbol{xy})$ **(N/D-20)(A/M-19) (M/J-16)**
9. **Solve:-** $(\boldsymbol{y} - \boldsymbol{xz})\boldsymbol{p} + (\boldsymbol{yz} - \boldsymbol{x})\boldsymbol{q} = (\boldsymbol{x} + \boldsymbol{y})(\boldsymbol{x} - \boldsymbol{y})$
10. **Solve:-** $(\boldsymbol{z^2} - \boldsymbol{y^2} - \boldsymbol{2yz})\boldsymbol{p} + (\boldsymbol{xy} + \boldsymbol{zx})\boldsymbol{q} = \boldsymbol{xy} - \boldsymbol{zx}$ **(N/D-15) (A/M-17)**

II- solve the partial diff-equations

1. Find the S.I $Z = px + qy + \sqrt{1 + p^2 + q^2}$ **(M/J-16)**
2. Find the S.I $Z = px + qy + p^2 - q^2$ **(N/D-14)**

3. Find the S.I $Z = px + qy + p^2q^2$ **(A/M-15) (N/D-20)**

4. Find the C.I $z^2(p^2 + q^2) = (x^2 + y^2)$ **(N/D-15)**

5. Find the C.I $p^2 + x^2y^2q^2 = x^2z^2$ **(N/D-20)**

6. Find the general solution $Z = px + qy + p^2 + pq + q^2$ **(A/M-18) (A/M-17)**

7. Solve :- $\sqrt{p} + \sqrt{q} = 1$

8. Solve :- $p^2 + x^2y^2q^2 = x^2z^2$ **(A/M-15)**

9. Solve :- $x^2p^2 + y^2q^2 = z^2$

10. Solve :- $p(1 + q) = qz$

SECOND HALF (8-MARKS)

III – Solve the homogeneous and non- homogeneous equations

1. Solve:- $\left(D^3 - 7DD'^2 - 6D'^3\right)z = \sin(x + 2y)$ **(N/D-14)**
2. Solve:- $\left(D^3 + D^2D' + 4DD'^2 + 4D'^3\right)z = \cos(2x + y)$
3. Solve:- $\left(D^2 + D'^2\right)z = x^2y^2$ **(N/D-15)**
4. Solve:- $\left(D^2 + 2DD' + D'^2\right)z = 2\cos y - x\sin y$ **(N/D-15)**
5. Solve:- $\left(D^2 + DD' - 6D'^2\right)z = y\cos x$ **(A/M-18)(N/D-16)**
6. Solve:- $\left(D^2 - 5DD' + 6D'^2\right)z = y\cos x$ **(N/D-17)**
7. Solve:- $(D^2 - 2DD')z = x^3y + e^{2x-y}$ **(N/D-14)**
8. Solve:- $\left(D^2 + 4DD' - 6D'^2\right)z = sin(x - 2y) + e^{2x-y}$ **(A/M-18)**
9. Solve: $(D^3 - 2D^2D')z = 2e^{2x} + 3x^2y.$ **(M/J-16)**
10. Solve:- $\left(D^2 + DD' - 6D'^2\right)z = x^2y + e^{3x+y}$
11. Solve:- $\left(D^2 + 2DD' + D'^2\right)z = x^2y + e^{x-y}$ **(A/M-17)**

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12. Solve:- $\left(D^2 + 2DD' + D'^2\right)z = e^{x-y} + xy$

13. Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y.$ **(A/M-19)**

14. Solve:- $\left(D^2 - 3DD' + 2D'^2 + 2D - 2D'\right)z = \sin(2x + y)$ **(M/J-16) (A/M-17)**

PART A

1. Form the partial differential equation by eliminating the arbitrary constants 'a' and 'b' from $\boldsymbol{log(az - 1) = x + ay + b}$ **(A/M-15)**
2. Construct the partial differential equation of all spheres whose centers lie on the Z-axis by the elimination of arbitrary constants. **(N/D-20)(N/D-15)**
3. Form the pde from the equation $2z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$. **(A/M-19)**
4. Form the partial differential equation by eliminating the arbitrary functions from $\boldsymbol{f(x^2 + y^2, z - xy) = 0}$. **(M/J-16) (N/D-18)**
5. Form the PDE by eliminating the arbitrary functions from $\boldsymbol{z = f(x^2 - y^2)}$.
6. Find the PDE of all spheres whose centers lie on the X-axis. **(N/D-16)**
7. Form the partial differential equation by eliminating the arbitrary constants 'a' and 'b'from $\boldsymbol{z = ax^2 + by^2}$. **(A/M-17)**
8. Form the PDE by eliminating the arbitrary functions from $\boldsymbol{z = f(y/x)}$. **(N/D-14)**
9. Form the partial differential equation by eliminating the arbitrary functions from $\emptyset(\boldsymbol{x^2 + y^2 + z^2, ax + by + cz}) = \boldsymbol{0}$.
10. Form the PDE by eliminating the arbitrary functions from $\boldsymbol{z = xf(2x + y) + g(2x + y)}$.
11. Find the complete solution of $\boldsymbol{q = 2px}$ **(A/M-15)**
12. Find the complete solution of $\boldsymbol{p + q = 1}$. **(N/D-14)**
13. Find the complete integral of $\boldsymbol{\frac{z}{pq} = \frac{x}{p} + \frac{y}{q} + \sqrt{pq}}$. **(N/D-16)**
14. Find the complete integral of $Z = px + qy + \sqrt{pq}$ **(A/M-18)**

15. Find the complete solution of PDE $\boldsymbol{p^3 - q^3 = 0}$. **(M/J-16)**

16. Find the complete integral of $\boldsymbol{pq = xy}$.

17. Solve $\frac{\partial^2 z}{\partial x \partial y} = 0$. **(A/M-17)**

18. Solve $(D^3 - 3DD'^2 + 2D'^3)z = 0$ **(A/M-18)**

19. Solve$(D + D' - 1)(D - 2D' + 3)z = 0$. **(N/D-20) (N/D-15)**

20. Solve $(D^3 - D^2D' - 8DD'^2 + 12D'^3)z = 0$

21. Solve $z = 1 + p^2 + q^2$.

22. Solve $(D^2 - DD' + D)z = 0$.

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UNIT II – FOURIER SERIES

PART-B

FIRST HALF (8-MARKS)

I- Find the ODD and EVEN type in the interval $[-\pi, \pi]$ and $[-l, l]$

1. **Find the Fourier series $f(x) = x^2$, $in -\pi < x < \pi$. Hence deduce the value of** $\sum_{n=1}^{\infty} \frac{1}{n^2}$.
2. $f(x) = x^2$, $in\ the\ interval$ $[-\pi, \pi]$ and deduce that
   (i) $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots \ldots \ldots = \frac{\pi^2}{6}$ (ii) $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots \ldots .. = \frac{\pi^2}{12}$
   (iii) $1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots = \frac{\pi^2}{8}$ **(N/D-19)**
3. Expand $f(x) = x^2$ as a Fourier series in the interval ($-\pi, \pi$) and
   Hence deduce that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots \ldots \ldots \ldots .. = \frac{\pi^4}{90}$ **(N/D-20)** **(N/D-16)**
4. Find the Fourier series of $f(x) = x + x^2$ $in -\pi < x < \pi$ of periodicity of 2π **(N/D-17)**
5. Find the Fourier series of $f(x) = |\sin x|$ $in -\pi < x < \pi$ of periodicity of 2π **(A/M-15)**
6. Find the Fourier series of $f(x) = |\cos x|$ $in -\pi < x < \pi$ **(M/J-16)**
7. Find the Fourier series expansion the following periodic function
   $f(x) = \begin{cases} 2 + x & , -2 < x < 0 \\ 2 - x & , 0 < x < 2 \end{cases}$ .Hence deduce that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots = \frac{\pi^2}{8}$ **(N/D-15)**
8. Find the Fourier series expansion the following periodic function

   $f(x) = \begin{cases} 1 - x & , -\pi < x < 0 \\ 1 + x & , 0 < x < \pi \end{cases}$ **(M/J-07)(N/D-13)**

II-Half-Range Series

(a) Find the Cosine Series

1. Find the half range cosine series of $\mathbf{f(x) = (\pi - x)^2, 0 < x < \pi}$.
   Hence find the sum of the series $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots \ldots \ldots \ldots$ **(N/D-15)**
2. Find the half range cosine series of $\mathbf{f(x) = (x - 1)^2}$, **in** $\mathbf{0 < x} < 1$. **(N/D-14)**
3. Obtain the Fourier cosine series expansion of $\mathbf{f(x) = x(\pi - x)}$, **in** $\mathbf{0 < x < \pi}$. **(N/D-14)**
4. Obtain the Fourier cosine series expansion of $\mathbf{f(x) = x}$, **in** $\mathbf{0 < x < \pi}$.
   Hence deduce that value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots \ldots$ **(N/D-17)**
5. Expand $\mathbf{f(x)} = \begin{cases} \mathbf{x} & , \mathbf{0 < x} < 1 \\ \mathbf{2 - x} & , \mathbf{1 < x} < 2 \end{cases}$ as a series of cosine in the interval **(0,2)** **(N/D-20) (A/M-17)**
6. Find the half range cosine series of $\mathbf{f(x) = x \sin x}$, in the interval $[\mathbf{0}, \boldsymbol{\pi}]$ **(N/D-11)**

(b) Find the Sine series

1. Find the half- range sine series of $\mathbf{f(x)} = \begin{cases} \mathbf{x}, & \mathbf{0 \le x \le \frac{\pi}{2}} \\ \boldsymbol{\pi} - \mathbf{x}, & \mathbf{\frac{\pi}{2} \le x \le \pi} \end{cases}$.

Hence deduce the sum of the series $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$ **(A/M-15)**

2. Find the half-range sine series of $\mathbf{f(x) = x \cos \pi x}$, in the $[\mathbf{0}, \mathbf{1}]$ **(M/J-16)**

SECOND HALF (8MARKS)

III- Complex form of the fourier series

1. Find the complex form of the Fourier series of $f(x) = e^{-x}$ in $-1 < x < 1$. **(A/M-15)**
2. Find the complex form of the Fourier series of $f(x) = e^{-ax}$, *in the* $[-\pi, \pi]$ **(N/D-20)**
3. Find the complex form of the Fourier series of $f(x) = e^{ax}$, *in the* $[-\pi, \pi]$
   where 'a' is a real constant. Hence deduce that $\sum_{n=-\infty}^{\infty} \frac{\pi}{a \sin a\pi}$ **(N/D-15)**
4. Expand $f(x) = e^{-ax}, -\pi < x < \pi$ as the complex form Fourier series. **(N/D-16)**
5. Find the complex form of the Fourier series of $f(x) = e^{-ax}$ in , $-l < x < l$. **(A/M-17)**

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IV- Find the Fourier series in the interval (0,2l)

1. Find the Fourier series of period $2l$ for the function $f(x) = (l-x)^2, \mathbf{0 < x < 2}l$

   Deduce the sum $\sum_{n=1}^{\infty} \frac{1}{n^2}$

2. Find the Fourier series of period $2l$ for the function $f(x) = \begin{cases} l-x, 0 < x < l \\ 0 \quad , l < x < 2l \end{cases}$

V-Find the Fourier series in the interval $(0,2\pi)$

1. **Find the Fourier series for the function** $\boldsymbol{f(x) = (\pi - x)^2}$, $\boldsymbol{in\ interval\ 0 < x <} 2\boldsymbol{\pi}$.

   Deduce the sum $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots \ldots \ldots \ldots$

2. Find the Fourier series of period 2π for the function $\boldsymbol{f(x) = x\ cos\ x}$ **in** $\mathbf{0} < x < 2\pi$ **(A/M-17)**

VI- Find the Harmonic values

1. Compute up to the first three harmonics of the Fourier series of f(x) given by the following table.

   (A/M-15) (N/D-14) (N/D-17) (A/M-18)

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
f(x)	1	1.4	1.9	1.7	1.5	1.2	1.0

2. Determine the first two harmonics of Fourier series for the following data. **(N/D-15)**

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	
F(x)	1.98	1.30	1.05	1.30	-0.88	-0.25	

3. Obtain the constant term and the coefficient of the first sine and cosine terms in the Fourier expansion of Y as given in the table. **(N/D-20)(N/D-16) (A/M-17)**

x	0	1	2	3	4	5
f(x)	9	18	24	28	26	20

4. Find the Fourier cosine series up to third harmonic to the following data **(M/J-16)**

x	0	1	2	3	4	5
f(x)	4	8	15	7	6	2

PART-B

1. Write down Dirichlet's conditions of Fourier series. **(M/J-16)**
2. Expand $f(x) = 1, in\ (0,\pi)$ as a half range sine series. **(N/D-15)**
3. If the fourier series of the function f(x) = x, in $(-\pi < x < \pi)$ with period 2π is given by **(M/J-16)**
   $f(x) = 2(\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \frac{\sin 4x}{4} + \cdots)$ then find the sum of the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$ ...
4. Find the sin series function $f(x) = 1, in\ (0,2)$ **(N/D-13)**
5. Find the sin series function $f(x) = 1, in\ 0 \le x \le \pi$ **(A/M-17)**
6. Find the value of the fourier series of $f(x) = \begin{cases} 0\ in\ (-c,0) \\ 1\ in\ (0,c) \end{cases}$ at the point of discontinuity $x = 0$. **(N/D-15)**
7. Find the value of b_n in the Fourier series of $f(x) = \begin{cases} x+\pi\ in\ (-\pi,0) \\ -x+\pi\ in\ (0,\pi) \end{cases}$. **(N/D-14)**
8. State the sufficient condition for existence of Fourier series. **(A/M-17)**
9. If $(\pi - x)^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{\cos nx}{n^2}\ \ in\ 0 < x < 2\pi$,then deduce that value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ **(N/D-14)**
10. If the Fourier series of the function $f(x) = x + x^2$, in the interval $(0,\pi)$ is **(N/D-20)(A/M-14)**
    $\frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \left[\frac{4}{n^2} \cos nx - \frac{2}{n} \sin nx\right]$, then find the value of the infinite series $1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$ .....
11. Find the root mean square value of $f(x) = x(l - x)\ in\ 0 \le x \le l$. **(N/D-14)**
12. Definition of root mean square value (RMS value) of a function $f(x)\ in\ a < x < b$.
13. The cosine series for $f(x) = x \sin x$ for $0 < x < \pi$ is given as $x \sin x = 1 - \frac{1}{2} \cos x - 2 \sum_{n=2}^{\infty} \frac{(-1)^n}{n^2 - 1} \cos nx$.
    Deduce that $1 + 2\left[\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots\right] = \frac{\pi}{2}$. **(A/M-14)**
14. If $f(x) = x^2\ in\ (0,2\pi)$ find the value of a_0 in the Fourier series.
15. State Parseval's theorem on Fourier series.
16. Find the value of a_0 in the Fourier series of $f(x) = e^x\ in\ (0,2\pi)$
17. Determine the value of a_n in the fourier series of $f(x) = x^3\ in\ -\pi < x < \pi$.
18. Find a_n , if the Fourier series of $f(x) = x \sin x\ \ in\ 0 < x < \pi$
19. Write the formula for complex form of Fourier series in $(-l, l)$.
20. If $f(x) = x^2\ in\ (-l, l)$, find the value of a_0 in the Fourier series.
21. Expand $f(x) = k, in\ (0,\pi)$ as a half range sine series.

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UNIT III – APPLICATION OF PARTIAL DIFFERENTIAL EQUATIONS

PART-B (ALL ARE 16-MARKS)

FIRST HALF

TYPE-I (String with zero-velocity)

1. A string is stretched and fastened at two points $x = 0$ and $x = l$ motion is started by displacing the string into the form $\boldsymbol{y = k(lx - x^2)}$ from which is $t = 0$. Find the displacement of the time't'. **(A/M-15)**

2. A tightly stretched string of length $2l$ is fastened at both ends, the midpoint of the string is displaced by a distance $'h'$ transversely and the string is released from rest in this position. Find the displacement of the string at any time 't'. **(A/M-17)**

3. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $\boldsymbol{y(x, 0) = y_0 \ \ sin^3\left(\frac{\pi x}{l}\right)}$,If it is released from rest from this position. Find the displacement of the end time 't'.

4. A tightly stretched flexible string has its ends fixed at $x = 0$ and $x = l$. At time $t = 0$, The string is given a shape defined by $\boldsymbol{f(x) = kx^2(l - x)}$, where 'k' is a constant, and then released from rest. Find the displacement of any point 'x' of the string at any time $t > 0$.

5. Find the displacement of any point of a string, if it is of length $2l$ and vibrating between fixed end points with initial velocity zero $\boldsymbol{f(x)} = \begin{cases} \frac{kx}{l} & , \ \boldsymbol{in\ 0} < x < l \\ 2k - \frac{kx}{l} & , \boldsymbol{in\ l} < x < 2\boldsymbol{l} \end{cases}$

6. A tightly stretched string with fixed end points $x = 0 \ and \ x = l$ is initially in a position given by $\boldsymbol{y(x, 0) = k \ \sin\frac{3\pi x}{l} \cos\frac{2\pi x}{l}}$. If it is released from rest from this position, determine the displacement $y(x, t)$.

TYPE-II ***(*****String with non-zero velocity*****)***

1. If a string of length 'l' is initially at rest in its equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = \boldsymbol{V_0 sin^3}\left(\frac{\pi x}{l}\right), (\boldsymbol{0} < x < l)$. Determine the displacement function $y(x, t)$ at any time 't'.

(N/D-14)

2. Find the displacement of a string stretched between two fixed points at a distance of $2l$ apart when the string is initially at rest in equilibrium position and points of the string are given initial velocity V, where $\boldsymbol{V} = \boldsymbol{f(x)} = \begin{cases} \frac{x}{l} & , \ \boldsymbol{in\ 0} < x < l \\ \frac{(2l-x)}{l} & , \boldsymbol{in\ l} < x < 2\boldsymbol{l} \end{cases}$, x being the distance from an end point. **(A/M-16)**

3. If a string of length 'l' is initially at rest in its equilibrium position and each of its points is given the velocity V. Such that $= \begin{cases} \frac{2kx}{l} & , \boldsymbol{0} < x < \frac{l}{2} \\ \frac{2k(l-x)}{l} & , \frac{l}{2} < x < l \end{cases}$. Find the displacement function $y(x, t)$ at any time 't'. **(N/D-20)**

4. A tightly stretched string of length 'l' with fixed end points is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity $\boldsymbol{y_t(x, 0) = v_0 \ \sin\left(\frac{3\pi x}{l}\right) \cos\left(\frac{\pi x}{l}\right)}$. Where $0 < x < l$. Find the displacement of the string at a point, at a distance x from one at any instant 't' **(N/D-16)**

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5. A tightly stretched string with fixed end points $x = 0$ *and* $x = l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $V = \lambda x(l - x)$ then, Show that $\boldsymbol{y(x,t)} = \frac{8\lambda l^3}{\pi^4} \sum_{n=1,3,5}^{\infty} \frac{1}{n^4} \sin\left(\frac{n\pi x}{l}\right) \sin\left(\frac{n\pi at}{l}\right)$.

6. If a string of length 'l' is initially at rest in its equilibrium position and each of its points is given the velocity V. Such that $= \begin{cases} kx & , 0 < x < \frac{l}{2} \\ k(l - x), & \frac{l}{2} < x < l \end{cases}$. Find the displacement function $y(x,t)$ at any time't'.

(N/D-15)

7. A string is stretched between two fixed points at a distance $2l$ apart and the points of the string are given initial velocities V, where $\boldsymbol{V = f(x)} = \begin{cases} \frac{cx}{l} & , \text{ in } 0 < x < l \\ \frac{c}{l}(2l - x) & , \text{ in } l < x < 2l \end{cases}$, x being the distance from an end point. Find the displacement of the string at any time.

SECOND HALF

ONE DIMENSIONAL HEAT-EQUATION

1. A bar 10 cm long with insulated sides has its ends A and B maintained at temperature at $50^0 c$ *and* $100^0 c$, respectively, until steady state conditions prevails. The temperature at A is suddenly raised to $90^0 c$ and at the same time lowered to $60^0 c$ at B. Find the temperature distributed in the bar at time 't.' **(N/D-15)**

2. A long rectangular plate with insulated surface is l cm wide. If the temperature along one short edge is $(\boldsymbol{x}, \boldsymbol{0}) = (\boldsymbol{lx} - \boldsymbol{x^2})\boldsymbol{r}$ $\boldsymbol{0} < x < l$, while the other two long edges $x = 0$ *and* $x = l$ as well as the other short edge are kept at $0^0 c$. Find the steady state temperature function $u(x, y)$

3. A rod 30cm long has its ends A and B kept at $20^0 c$ *and* $80^0 c$ respectively until steady state conditions previl the temperature at each end its them suddenly reduced at $0^0 c$ and kept so. Find the resulting temperature function (x, t) taking x=0 at A.

4. The ends A and B of a rod 30cm long have their temperature at $20^0 c\ and\ 80^0 c$ until steady state conditions prevail. The temperature of the end B is suddenly reduced to $60^0 c$ and kept so while the end A is raised to $40^0 c$. Find the temperature distribution in the rod after time.

TWO DIMENSIONAL HEAT-EQUATION

1. A square plate is bounded by the lines $\boldsymbol{x = 0, x = a\ and\ y = 0, y = b}$. Its surfaces are insulated and the temperature along $y = b$ is kept at $100^0 c$. While the temperature along other three edges are at $0^0 c$. Find the steady state temperature at any point in the plat **(N/D-14)**

2. A square plate is bounded by the lines $\boldsymbol{x = 0, x = 20\ and\ y = 0, y = 20}$. Its faces are insulated. The temperature along the upper horizontal edge is given by $(\boldsymbol{x, 20})(\boldsymbol{20 - x})$, $\boldsymbol{0} <\ < 20$, while the other three edge are kept at $0^0 c$. Find the steady state temperature distribution (x, y) in the plate. **(N/D-16)**

3. A square plate is bounded by the lines $\boldsymbol{x = 0, x = 10\ and\ y = 0, y = 10}$. Its faces are insulated. The temperature along the upper horizontal edge is given by $(\boldsymbol{x, 0}) = (\boldsymbol{10 - x})$,While the other three edge are kept at $0^0 c$. Find the steady state temperature distribution (x, y) in the plate.

4. Along rectangular piate with insulated surface is l cm wide. If the temperature along one short edge i $(\boldsymbol{x, 0}) = (\boldsymbol{lx - x^2})\boldsymbol{r}\ \boldsymbol{0} < x < l$ s, while the other two long edges $x = 0\ and\ x = 1$ as well as the other short edge are kept at $0^0 c$, find the steady state temperature function $u(x, y)$

PART A

1. State the assumptions in deriving one-dimensional wave equation. `**(N/D-20)** **(N/D-16)**
2. State the three possible solution of the one-dimensional wave equation. **(A/M-14)**
3. State the three possible solution of the one-dimensional heat equation $u_t = \alpha^2 u_{xx}$. **(N/D-20)**
4. Classify the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + 10(x^2 + y^2 - 10) = 0$.
5. Classify the equation $(1 - x^2)z_{xx} - 2xyz_{xy} + (1 - y^2)z_{yy} + xz_x + 3x^2yz_y - 2z = 0$. **(N/D-14)**
6. Write down the various possible solution of the one-dimensional heat equation. **(M/J-16)**
7. A rod 30cm long has its ends A and B kept at 20c and 80c respectively until steady state conditions prevail.find the steady state temperature in the rod. **(A/M-14)**
8. Classify the equation $u_{xx} + u_{xy} = f(x, y)$. **(M/J-16)**
9. Write all possible solution of the two-dimensional heat equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. **(N/D-15)**
10. Solve $3x\frac{\partial u}{\partial x} - 2y\frac{\partial u}{\partial y} = 0$ using method of separation of variables. **(N/D-15)**
11. What is the constant α^2 in the wave equation.
12. In the wave equation $\frac{\partial^2 x}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$ what does c^2 stand for ?
13. What is mean by steady state condition in heat flow?
14. In steady state conditions derive the solution of one dimensional heat flow?
15. Difference between one dimensional wave and heat flow equations?
16. The PDE of one dimensional heat equation is $\frac{\partial u}{\partial t} = \alpha^2\frac{\partial^2 u}{\partial x^2}$. what is α^2?
17. What are the assumptions made in deriving one-dimensional heat equation?
18. State one-dimensional heat equation with the initial and boundary conditions?
19. State one-dimensional wave equation (zero initial velocity) with the initial and boundary conditions?
20. A rod 10cm long has its ends A and B kept at 20c and 70c respectively until steady state conditions prevail. find the steady state temperature in the rod

QUESTION BANK

PERIOD: JULY - NOV 2020 **BATCH**: 2018 – 2022

BRANCH: EEE **YEAR/SEM:** II/03

SUB CODE/NAME: MA8353 - TRANSFORM AND PARTIAL DIFFERENTIAL EQUATIONS

UNIT IV – FOURIER TRANSFORM

PART-B

FIRST HALF (16-MARKS)

I-Find the Fourier transform, Inversion and Parseval's identity of the function

1. Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$. Hence, deduce the values

$(i)\quad \int_0^\infty \frac{\sin t - t\cos t}{t^3}\, dt = \frac{\pi}{4} \qquad (ii)\ \int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt = \frac{\pi}{15}$

2. Find the Fourier transform of the function $f(x) = \begin{cases} a - |x|, & |x| < a \\ 0, & |x| > a \end{cases}$. Hence deduce that

$(i)\ \int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2} \qquad (ii)\ \int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$

3. Find the Fourier transform of the function $f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$. Hence deduce that

$(i)\ \int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2} \qquad (ii)\ \int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$ ***(N/D-14) (N/D-15) (N/D-16)***

4. Find the Fourier transform of $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| \geq a \end{cases}$ and hence evaluate $(i) \int_0^\infty \frac{\sin t}{t}\, dt.$

Using Parseval's identity, prove that $(ii)\ \int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$ ***(A/M-15)***

1. Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} 1, & |x| < 2 \\ 0, & |x| \geq 2 \end{cases}$ and hence evaluate

$(i) \int_0^\infty \frac{\sin x}{x}\, dx$ **and** $(ii)\ \int_0^\infty \left(\frac{\sin x}{x}\right)^2 dx$ ***(N/D-20)*****(A/M-17)**

5. Find the Fourier transform of $f(x) = \begin{cases} a, & |x| < 1 \\ 0, & |x| \geq 1 \end{cases}$ and hence evaluate $(i) \int_0^\infty \frac{\sin t}{t} dt = \frac{\pi}{2}.$

Using Parseval's identity, prove that $(ii) \int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$

SECOND HALF (8-MARKS)

II- *Using "sine" and "cosine" transform*

2. Find the Fourier cosine transform of the function $f(x) = \frac{e^{-ax} - e^{-bx}}{x}$ $, x > 0$ ***(N/D-15)***
3. Find the Fourier sine transform of the function $f(x) = \frac{e^{-ax} - e^{-bx}}{x}$ $, x > 0$
4. Find the Fourier cosine transform of x^{n-1}. ***(A/M-15)***
5. Find the Fourier cosine transform of $e^{-a^2x^2}$, for any$(a > 0)$ ***(N/D-20)***
6. Find the infinite Fourier sine transform of $f(x) = \frac{e^{-ax}}{x}$ hence deduce the infinite Fourier sine transform of $\frac{1}{x}$. ***(N/D-16)***
7. Find the fourier sine and cosine transform of a function $f(x) = e^{-ax}$. Using parseval's identity, evaluate $(i) \int_0^\infty \frac{dx}{(x^2+1)^2}$ and $(ii) \int_0^\infty \frac{x^2 dx}{(x^2+1)^2}$ (N/D-17)
8. Evaluate $\int_0^\infty \frac{dx}{(x^2+a)^2}$ using Fourier transform.
9. Evaluate $\int_0^\infty \frac{x^2 dx}{(x^2+a)^2}$ using Fourier transform.

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10. Evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ using Fourier transform. ***(N/D-14)***

11. Evaluate $\int_0^\infty \frac{dx}{(x^2+1)(x^2+4)}$ using Fourier transform. ***(N/D-20)(A/M-17)***

12. Evaluate:- $\int_0^\infty \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$ using Fourier transform

III- ***<u>Using self –reciprocal</u>***

1. Find the Fourier transform of $f(x) = e^{\frac{-x^2}{2}}$ in $(-\infty,\infty)$ ***(M/J-16)***

(or)

Show that the transform $e^{\frac{-x^2}{2}}$ is self reciprocal of $e^{\frac{-s^2}{2}}$

2. Find the Fourier transform of $e^{-a^2x^2}, (a > 0)$. Hence show that $e^{\frac{-x^2}{2}}$ is self reciprocal. ***(N/D-14)***

(or)

Find the infinite Fourier transform of $e^{-a^2x^2}, (a > 0)$. Hence deduce the infinite Fourier transform of $e^{\frac{-x^2}{2}}$ ***(N/D-16) (A/M-15) (A/M-17)***

<u>PART-A</u>

1. Find the Fourier sin transform of $\boldsymbol{f(x) = \frac{1}{x}}$. **(N/D-20) (A/M 2015, N/D 2016)**
2. If F(s) is the Fourier transform of f(x), Prove that $\boldsymbol{F\{f(x-a)\} = e^{ias} F(s)}$ **(A/M-17) (N/D-20)**
3. State Fourier integral theorem. **(M/J 16)**
4. Define Fourier transform pair. **(N/D 2011,10)**
5. State and prove modulation property on fourier transform. **(N/D 2014 ,13)**
6. If $\boldsymbol{F\{f(x)\} = F(s), then\ find\ F\{e^{iax} f(x)\}}$ **(A/M-15)**
7. State change of scale property on Fourier transforms. **(N/D-16)**
8. Find the Fourier transform of a derivative of the function f(x) if **f(x)→0 as x→** $\pm\infty$. **(M/J 2016)**
9. State convolution theorem on fourier sin transform. **(A/M 2017)**
10. Find the Fourier transform of f(x) = $\begin{cases} \mathbf{1}\ \boldsymbol{for}\ |\boldsymbol{x}| < a \\ \mathbf{0}\ \boldsymbol{for}\ |\boldsymbol{x}| > a \end{cases}$ **(A/M 2017)**

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11. If F(s) is the Fourier transform of f(x) , prove that $\boldsymbol{F\{f(ax)\} = \frac{1}{a}\, F\left(\frac{s}{a}\right), a \neq 0}$ **(N/D 2015)**

12. Evaluate $\int_0^\infty \frac{s^2}{(s^2+a^2)\,(s^2+b^2)}\, ds$ using fourier transforms.

13. Find the Fourier sine transform $f(x) = e^{-x}$

14. Find the Fourier cosine transform of $\boldsymbol{e^{-ax}}$ **, a > 0.**

15. Find the Fourier sine transform of $\boldsymbol{e^{-ax}}$**, a > 0.**

16. State convolution theorem on Fourier transforms.

17. State the parseval's identity on Fourier transform. **(N/D 2010,2011,2012)**

18. State and prove shifting property on fourier transform. **(N/D 2010,2013,2014,2015)**

19. Define fourier sine and its inverse transform.

20. Define fourier cosine and its inverse transform..

21. Prove that $\boldsymbol{F_s[x\, f(x)] = -\frac{d}{ds}[F_c(s)]}$.

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BRANCH: EEE **YEAR/SEM:** II/03

SUB CODE/NAME: MA8353 - TRANSFORM AND PARTIAL DIFFERENTIAL EQUATIONS

UNIT V – Z -TRANSFORM AND DIFFRENCE EQUATION

PART-B (8-MARKS)

FIRST HALF

I-Using Convolution Theorem

1. Using convolution theorem, evaluation $Z^{-1}\left[\frac{Z^2}{(Z-a)^2}\right]$ **(A/M-16)**
2. Using convolution theorem and evaluation $Z^{-1}\left[\frac{Z^2}{(Z-3)(Z-4)}\right]$ **(N/D-15)**
3. Using convolution theorem, find $Z^{-1}\left[\frac{Z^2}{\left(Z-\frac{1}{2}\right)\left(Z-\frac{1}{4}\right)}\right]$ **(A/M-15)**
4. Using convolution theorem, evaluate $Z^{-1}\left[\frac{Z^2}{(Z-a)(Z-b)}\right]$ **(N/D-16)**
5. Using convolution theorem, evaluate $Z^{-1}\left[\frac{Z^2}{(Z+a)(Z+b)}\right]$
6. Using convolution theorem, evaluate $Z^{-1}\left[\frac{8Z^2}{(2z-1)(4Z+1)}\right]$ **(N/D-20)(A/M-17)**
7. Using convolution theorem, find $Z^{-1}\left[\frac{Z^2}{\left(Z-\frac{1}{2}\right)\left(Z+\frac{1}{4}\right)}\right]$

II-Using Partial Fraction

1. Find the inverse Z-transform of $Z^{-1}\left[\frac{Z^2+Z}{(Z-1)(Z^2+1)}\right]$ *using partial fraction* **(N/D-14)**
2. Find the inverse Z-transform of $Z^{-1}\left[\frac{Z}{(Z-1)(Z^2+1)}\right]$ *using partial fraction* **(N/D-20)**
3. Find $Z^{-1}\left[\frac{4Z^3}{(2z-1)^2(z-1)}\right]$, by the method of partial fractions **(A/M-17)**
4. Find the inverse Z-transform of $\frac{z^3}{(z-1)^2(z-2)}$ by method of partial fraction **(N/D-17)**

III- Using z-transform

1. Find the Z-transforms of $\cos\frac{n\pi}{2}$ *and* $\frac{n}{n(n+1)}$ **(A/M-16)**

2. Find $Z[\, r^n \cos n\theta]$ and $Z[\, r^n \sin n\theta]$

5. Find (i) $Z[n^3]$ and (ii) $Z[e^{-t}t^2]$ **(N/D-16)**

6. Find (i) $Z[\cos n\theta]$ and (ii) $Z[\sin n\theta]$ **(N/D-14)**

SECOND HALF

IV-Using Residue Theorem

1. Find the inverse Z-transform of $\frac{Z}{Z^2-2Z+2}$ by residue method. **(A/M-15)**
2. Using the inversion integral method (Residue theorem), find the inverse Z-transform of $u(z) = \frac{z^2}{(Z+2)(z^2+4)}$ **(N/D-15)**
3. Using residue method, find $Z^{-1}\left[\frac{Z}{Z^2-2Z+2}\right]$ **(A/M-16)**
4. Evaluate $Z^{-1}\left[\frac{9Z^3}{(3Z-1)^2(z-2)}\right]$, using calculus of residues. **(N/D-16)**

V- Solve the difference equation using Z-transformations

1. Solve the difference equation $y_{n+2} + y_n = 2\ ,\ given\ that\ y_0 = 0\ and\ y_1 = 0$ bu using Z-transforms. **(A/M-16)**
2. Using the Z-transforms solve the difference equation $y(n+3) - 3y(n+1) + 2y(n) = 0\ , with\ \ y(0) = 4, y(1) = 0, y(2) = 8$ **(N/D-20)**
3. Using the Z-transforms solve the difference equation $u_{n+2} + 4u_{n+1} + 3u_n = 3^n\ , given\ that\ \ u_0 = 0\ and\ u_1 = 1$ **(N/D-15)**
4. Using the Z-transforms solve the difference equation $u_{n+2} - 4u_{n+1} + 4u_n = 0\ , given\ that\ \ u_0 = 1\ and\ u_1 = 0$ **(N/D-15)**
5. Using the Z-transforms solve the difference equation $x_{n+2} - 3x_{n+1} + 2x_n = 0\ , given\ that\ \ x_0 = 0\ and\ x_1 = 1$ **(N/D-14) (A/M-15)**
6. Using the Z-transform, solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n\ ,\ given\ that\ \ y_0 = 0\ and\ y_1 = 0$ **(N/D-16)**
7. Using the Z-transform, solve the difference equation $y_{n+2} - 7y_{n+1} + 12y_n = 2^n\ ,\ given\ that\ \ y_0 = 0\ and\ y_1 = 0$ **(N/D-17)(A/M-15)**
8. Using the Z-transform, solve the difference equation $y_{n+2} + 4y_{n+1} + 3y_n = 2\ , given\ that\ y_0 = 0\ and\ y_1 = 1$

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PART-A

1. Find the Z-transform of {n}. **(N/D-14)**
2. Find the Z-transform of $F(n) = \frac{1}{n}$. **(N/D-17)**
3. State the initial value theorem on Z-transform. **(N/D-14) (N/D-15) (A/M-17)**
4. State the final value theorem on Z-transform. **(N/D-20)**
5. State convolution theorem on Z-transform. **(N/D-16) (A/M-15)**
6. Prove that $Z\{nf(n)\} = -z\frac{d}{dz}F(z), where\ z\{f(n)\} = F(z)$ **(A/M-18)**
7. Find $Z\left[\frac{1}{n(n+1)}\right]$ **(N/D-16) (N/D-15)**
8. Find $Z\left[\frac{1}{n!}\right]$ **(M/J-16)**
9. Find $Z[(\cos\theta + i\sin\theta)^n]$ **(M/J-16)**
10. $If\ Z(x(n)) = X(n)$,then show that $Z[a^n x(n)] = X\left(\frac{z}{a}\right)$
11. Find the Z-transform of a^n . **(A/M-17)**
12. Find $\boldsymbol{Z}[\boldsymbol{a^n}]$
13. State the final value theorem on Z-transform.
14. Form the difference eqn by elimination arbitrary constant ‘a’ from $Y_n = a.2^n$
15. Find $Z\left[\frac{a^n}{n!}\right]$
16. Find Z[cos$\frac{n\pi}{2}$]
17. Define:- unit impulse function
18. Find $Z(-1)^n$
19. Find Z[cos nθ]
20. Find $Z[\sin\ n\theta]$